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Theory and Methodology

Determining optimal pool size of a temporary Call-In work force

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Abstract: This paper is one in a series that introduces concepts of JUST-IN-TIME-PERSONNEL. Management of worker job time and assignment are in many ways analogous to inventory management. Idle workers represent unutilized, 'inventoried' personnel, imposing potentially large costs on management. But a lack of workers when needed may force the use of otherwise unnecessary overtime or other emergency procedures, creating excessive costs analogous to costs of stockout in traditional inventory systems. A system having JUST-IN-TIME-PERSONNEL attempts to meet all demands for personnel at minimum cost by sharply reducing both excess worker inventory with its concomitant 'paid lost time' and underage of worker inventory with its associated costs of stockout. The model in this paper focuses on one important component of a JUST-IN-TIME or 'JIT' PERSONNEL system: response to day-to-day fluctuations in workload, worker outages due to sick leave, personal constraints or other unscheduled events. To maximize utilization of the JIT concept, we assume there exists a pool of call-in personnel who can be called on the day that they are needed. Each such call-in 'temp' is guaranteed a minimum number of offered days per month. A temp is paid each month for the days actually worked plus the differential, if any, between the number of days offered and the number of days guaranteed. Temps, like regular workers, may be unavailable on any given day due to illness, etc. The analysis leads to an exact probabilistic model that can be solved to find the optimal pool size of temps. Numerical results are included.

Keywords: Work force management; Optimal pool size; Temporary work force

1. Background and assumptions

In many industries the largest single variable cost is cost of personnel. While extensive analyses have been performed on the logistics and technologies of a firm's operations, including the just-in-time

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delivery of materials, relatively little effort has been devoted to cost-effective flexible scheduling and job assignments of personnel. There are numerous reasons for focusing on efficiency and flexibility in personnel scheduling: (1) sick days; (2) time-of-day, day-of-week and season-of-year predictable changes in demand for personnel; (3) probabilistic changes in demands for personnel; (4) breakdowns in production technology which might require a greater or lesser number of personnel present.

Implementation of a JUST-IN-TIME-PERSONNEL system means having operating procedures in place to schedule efficiently for predictable demand profiles. But it also means having often untraditional procedures in place to respond to stochastic changes in demand for personnel and/or personnel availability (e.g., due to sick leave).

The model developed in this paper focuses on response to worker outages due to sick leave, personal constraints or other unscheduled events. Our focus is on a workforce comprising N full time employees (FTE's) and M temporary 'call-in' employees. We assume that each day there are D full shifts of work to be done, each shift totaling S hours of work. (D can fluctuate from day to day). Also, due to sickness, random absenteeism, training, military duties, etc., not all of the FTE's may appear for work on a given day. The shifts not staffed by regular FTE's must be filled by call-in temps and/or overtime shifts of FTE's. The cost per hour of a temp is less than that of overtime, so the preference is to utilize the temps first before resorting to overtime.

To induce the temps to sign up for such part time call-in work, management guarantees that a certain minimum number of work days (shifts) will be offered to each temp each month. If less than the guaranteed amount of days is offered, the temp will be paid for the 'missing gap of shifts', i.e., for the difference between the guaranteed amount and the offered amount. In addition, each temp will be paid for each offered day that is worked. It is assumed that due to sickness, personal plans, etc., a certain fraction of time that a temp is offered work (s)he is unable to accept it (in which case (s)he will not be paid for that day). To assure equity in the distribution of offered days, the workforce manager when calling temps proceeds serially down the list of temps, numbered from 1 to M, always starting on a given day immediately following the last temp called on the most recent day that temps were called and recycling to temp number 1 after calling temp number M. This process guarantees that at any given time during the month the difference in the numbers of days offered to any two temps cannot exceed one.

In structuring this two-tiered workforce there are four costs: (1) the fixed cost of the N full time employees; (2) the variable cost of the shifts worked by temps; (3) the variable cost of overtime; and (4) the variable cost associated with paying temps for shifts not worked due to a shortfall in the monthly guaranteed number of offered shifts. The system objective is to 'size' the pool of temps so that expected total costs of the workforce are minimized. That is, we wish to find the optimal value of M so that total expected costs are minimized. If M is too small, the costs of overtime will be excessive. If M is too large, the cost of paying temps for time not worked dominates. The objective is to find that M that optimally balances the tradeoffs between these two competing cost factors.

The motivation for our model is derived from operations of postal services. For instance, consider a local post office. On any given day the post office will receive a random quantity of mail that must be 'final sorted' to route delivery sequence and then delivered to postal customers on each route. Statistical studies have shown that the quantity of mail received on any given day can be accurately modeled as a normal or Gaussian random variable with mean and variance known from historical data. The amount of postal work to be done at the sorting station is linearly related to the quantity of mail received and is thus a normal random variable. A certain number of full time postal workers are scheduled to work each day, but one, two, or more of them may be absent due to illness or other reasons. Since the mail must be delivered on the day received at the local post office, available management options prior to our model were (1) use of overtime and/or (2) use of scheduled full time 'fill-in' workers. This latter category of worker has no regular assignment but appears for work in anticipation of 'being a substitute' for an absent worker regularly scheduled for the mail sorting (and delivery) assignment. Use of such full time paid substitutes is potentially inefficient and costly; often they may appear for work and find little work to perform. Both the overtime and the fill-in personnel options had been used in the postal facilities studied. Our model, utilizing call in temporary personnel, identified and analyzed a new operational

Table 1
Glossary of terms

N	The total number of full time employees or FTE's.
X	Random variable (r.v.) corresponding to number of FTE's who appear for work on a random day.
p	The probability that a random FTE does not appear for work on any given day.
S	Shift length (in hours) for any worker (FTE or call-in).
M	The number of people ('temps') in the call-in pool of workers.
G	The guaranteed minimum number of days per month that each temp will be offered work.
p_1	Probability that a random temp will not accept offered work on a random day.
A	The total number of temps (from the pool of M) who are willing to work on a random day.
C_R	Hourly cost of an FTE.
C_{OT}	Hourly cost of overtime work performed by FTE's
C_{TE}	Hourly cost of a temp.
T	r.v. corresponding to the number of FTE's who are absent on any given day.
S_k	r.v. corresponding to number of phone calls made to temps until k of them accept work assuming an infinite pool of temps.
OT	r.v. corresponding to number of overtime shifts to be worked on a random day.
TE	Expected number of temps who show up for work on a random day.
V	Total number of work days in a month.
Y	r.v. corresponding to total number of work-offering phone calls made to temps during a random month.
D	r.v. corresponding to the number of workers (shifts) required per day.

option that potentially could (1) reduce the use of overtime and (2) eliminate the need for costly and frequently unneeded fill-in personnel. Too, the availability of call in temporary assignments might draw into the labor force nontraditional types of workers, such as those partially limited by 'spousal hours' and other personal constraints that preclude their participation in the full time scheduled workforce. The identical model, with different parameter values, can be applied to other types of postal facilities, such as large 'mail sorting centers' often employing hundreds or even thousands of workers. Our analysis was presented to postal authorities but as of this writing no implementation can be reported.

The model can be applied in any operation having workload that varies on a day-to-day basis and operating under the constraint that work cannot be inventoried from one day to the next; that is, all work received during a given day must be processed during that day. Other (nonpostal) examples include bank check clearing facilities, certain insurance operations, certain customer service operations, certain emergency repair operations, etc.

The model herein is but one representation of the use of 'pooled' temporary workers in the work force. For a comprehensive taxonomy, additional models and a comprehensive set of additional references we recommended the Ph.D. thesis by Leegwater [2]. Also, for application of similar ideas in the urban transportation industry see Wilson [3].

2. The model

In this section we detail the assumptions and notation of the model. For reader convenience, a glossary of variables is provided in Table 1.

2.1. Absenteeism

In the model there are two types of absenteeism: due to FTE's and due to temps. For a random FTE (temp) we assume that (s)he will not be available for work on any given day with probability p (p_1, respectively). All workers (FTE's and temps) behave independently with regard to absenteeism, so that the number of FTE's (temps) who are available for work on any given day is a binomial r.v. with parameters $1-p$ and N ($1-p_1$ and M, resp.).

We let r.v. X (A) be the number of FTE's (temps) who are available and willing to work on a random day. Then the number of FTE's who are absent on a random day is $T = N - X$, and the number of temps not willing to work is $M - A$.

2.2. Demand for personnel

Daily demand D at the facility, measured in number of workers (shifts) required for the day is assumed to be a random variable with probability distribution $P(D = d)$. It is further assumed that the two random variables T and D are statistically independent.

2.3. Phone calls to temps

Suppose $T = i$ and $D = d$ on a given day, i.e., there are i absences in the full time work force and there is a demand for d workers. Clearly, if $k = d + i - N \leq 0$, there will be no need for temps. If $k \geq M$, all M temps will be called and A will report to work.

For all $k \geq 1$ we are interested in the number of temps called and the number who are willing to work. The workforce manager starts phoning temps in serial fashion, starting in the list immediately after the temp last called on the previous day (or two or more days earlier, if no temps were called on the previous day(s)). He continues these calls until either k temps have been found who are willing to work or until all M temps have been called.

Suppose for the moment that the pool of temps is infinitely large, i.e., $M = \infty$. Then, defining

$S_k \equiv$ r.v. corresponding to the total number of phone calls made on a random day having i absences and a demand of d $(k = d + i - N)$ in the full time work force, $k = 1, 2, \ldots,$

the r.v. S_k is simply the number of independent Bernoulli trials until the 'k-th success', assuming success probability on any one trial being $1 - p_1$. The probability law for S_k is Pascal,

$$P(S_k = n) = \binom{n-1}{k-1}(1 - p_1)^k p_1^{n-k}, \quad n = k, k+1, \ldots, \tag{1}$$

To convert this result back to a temp pool of size M, we focus on $n = M$. For instance, the probability that k temps will be found who are willing to work on a day having k absences in the full time work force is $P(S_k \leq M)$. The probability that only $j < k$ temps will be found on a given day is

$$\begin{aligned}\text{Prob}\{j \text{ temps willing to work} \mid k \text{ needed; } k > j\} &= P(S_{j+1} > M) - P(S_j > M) = P\{A = j\} \\ &= \binom{M}{j}(1 - p_1)^j p_1^{M-j}.\end{aligned} \tag{2}$$

2.4. Overtime shifts

If after calling all temps there is still unmet work requirements for today, then the remainder is provided by overtime shifts. Let

OT = r.v. corresponding to the number of overtime shifts required on a random day.

We first consider the conditional probability distribution of OT, given that the number of required temps T and the demand D are known. Let us define a new random variable $Z = T + D - N$. Obviously when Z is positive, it is the number of temps required per day and when $Z \leq 0$ no temps are required. Suppose L_1 and L_2 are respectively the minimum and maximum levels that D can assume. The PMF of Z is given by

$$P(Z = k) = \sum_{[(i,d); k = i + d - N]} P(D = d) \cdot P(T = i), \quad k = L_1 - N, \ldots, L_2. \tag{3}$$

Now $P(\mathrm{OT}=j \mid Z=k)$ can be expressed as follows, where

$$P(T=i)=\binom{N}{i}p^i(1-p)^{N-i}, \quad i=0,1,\ldots,N:$$

$$P\{\mathrm{OT}=j \mid Z=k\}=\begin{cases} 1, & j=0, \quad k\le 0,\\ P(S_k\le M), & j=0, \quad 1\le k\le M,\\ 0, & j=0,1,\ldots,k-M-1, \quad k>M,\\ P(S_M=M)=(1-p_1)^M, & j=k-M, \quad k>M,\\ P(S_{k-j}\le M)-P(S_{k+1-j}\le M) & \\ \quad =\binom{M}{k-j}(1-p_1)^{k-j}p_1^{M-(k-j)}, & 1\le k-M+1\le j<k,\\ 1-P(S_1\le M), & j=k, \quad k>0,\\ 0, & j>k, \quad k>0,\\ 0, & j>0, \quad k\le 0. \end{cases} \tag{4}$$

Now the unconditional probability distribution of OT can be written

$$P(\mathrm{OT}=j)=\sum_{k=L_1-N}^{L_2} P(\mathrm{OT}=j \mid Z=k)\ P(Z=k), \quad j=0,1,\ldots,L_2, \tag{5}$$

and the expected number of overtime shifts is

$$\overline{\mathrm{OT}}=E[\mathrm{OT}]=\sum_{j=0}^{L_2} jP(\mathrm{OT}=j). \tag{6}$$

To simplify (6) we introduce the following lemma:

Lemma 1.

$$E(\mathrm{OT} \mid Z=k)=\begin{cases} k-\sum_{l=1}^{M} P(S_l\le M), & k\ge M,\\ k-\sum_{l=1}^{k} P(S_l\le M), & k<M. \end{cases} \tag{$*$}$$

Remark. As the proof will show, the RHS of $(*)$ is just another way of expressing k minus the expected number of temps who are willing to work, given that k are required.

Proof. Suppose $k\ge M$. Then

$$\begin{aligned} E(\mathrm{OT} \mid Z=k) &= (k-M)P(S_M\le M)+(k-M+1)[P(S_{M-1}\le M)-P(S_M\le M)]\\ &\quad +(k-M+2)[P(S_{M-2}\le M)-P(S_{M-1}\le M)]\\ &\quad +\ldots+(k-1)[P(S_1\le M)-P(S_2\le M)]\\ &\quad +k[1-P(S_1\le M)]=k-\sum_{l=1}^{M} P(S_l\le M). \end{aligned}$$

Suppose $k < M$. Then

$$\begin{aligned} E(\mathrm{OT} \mid Z = k) = &[P(S_{k-1} \le M) - P(S_k \le M)] + 2[P(S_{k-2} \le M) - P(S_{k-1} \le M)] \\ &+ 3[P(S_{k-3} \le M) - P(S_{k-2} \le M)] \\ &+ \ldots + (k-1)[P(S_1 \le M) - P(S_2 \le M_2)] \\ &+ k[1 - P(S_1 \le M)] = k - \sum_{l=1}^{k} P(S_l \le M). \end{aligned}$$

□

Lemma 2. *$E(\mathrm{OT} \mid Z = k) = k - M(1 - p_1)$ for $k \ge M$.*

This result is obvious since, given $Z \ge M$, all M temps will be called, and an expected $M(1 - p_1)$ will be available for work.

Now $\overline{\mathrm{OT}}$ is given by

$$\begin{aligned} \overline{\mathrm{OT}} = E_k(E(\mathrm{OT} \mid Z = k)) &= \sum_{k=1}^{M-1} \left(k - \sum_{l=1}^{k} P(S_l \le M) \right) P(Z = k) + \sum_{k=M}^{L_2} (k - M(1 - p_1)) P(Z = k) \\ &= E \max(Z, 0) - M(1 - p_1) \sum_{k=M}^{L_2} P(Z = k) - \sum_{k=1}^{M-1} P(Z - k) \cdot \sum_{l=1}^{k} P(S_l \le M). \end{aligned} \tag{7}$$

The expected overtime cost during the month is

$$\overline{\mathrm{OT}}_{\mathrm{M}} = \overline{\mathrm{OT}} \cdot S \cdot V \cdot C_{\mathrm{OT}}. \tag{8}$$

Notice that (7) is quite intuitive. $\overline{\mathrm{OT}}$ is equal to the expected number of workers required minus the conditional expected number of temps who are willing to work, given that at least M temps are required, multiplied by the appropriate conditional probability, minus the conditional expected number of temps who agree to work, given that less than M temps are required, multiplied by the appropriate conditional probability.

The expected number of temps who appear for work on any given day is

$$\overline{\mathrm{TE}} = E \max(Z, 0) - \overline{\mathrm{OT}} = M(1 - p_1) \sum_{k=M}^{L_2} P(Z = k) - \sum_{k=1}^{M-1} P(Z = k) \sum_{l=1}^{k} P(S_l \le M) \tag{9}$$

and the expected monthly direct cost of temps is

$$\overline{\mathrm{TE}}_{\mathrm{M}} = \overline{\mathrm{TE}} \cdot S \cdot V \cdot C_{\mathrm{TE}}. \tag{10}$$

2.5. The cost of underutilized temps

The only expected cost component that remains to be developed is the indirect cost of temps that may occur because the number of days of work offered during a month is less than that guaranteed. Let W_l = Number of temps called in day l, $l = 1, 2, \ldots, V$.

Clearly the W_l's are mutually independent $l = 1, 2, \ldots, V$. Recalling that T is the number of temps absent on a given day and that the maximum possible number of calls is M, we can write

$$W_l = \min\left[M, \sum_{i=1}^{Z} v_i \right],$$

where the v_i's are i.i.d geometric r.v.'s each with mean $1/(1-p_1)$. Recall that the pmf for the sum of a fixed number of independent geometric r.v.'s is Pascal, i.e.,

$$P\left\{\sum_{i=1}^{k} v_i = j\right\} = \binom{j-1}{k-1}(1-p_1)^k (p_1)^{j-k}, \quad j=k, k+1,\ldots, \quad k=1, 2,\ldots,$$

and the pmf for Z is given by (3).

Combining the Pascal and (3) allows us to assign a probability to each lattice point in the W_l, Z sample space, i.e.,

$$P\{W_l = j,\ Z=k\} = P\{W_l = j \mid Z=k\}\ P\{Z=k\},$$

which is expressed as a product of the two respective pmf's. We then sum vertically over the lattice points to obtain the marginal pmf for W_l:

$$P\{W_l = j\} = \begin{cases} P(Z \le 0), & j=0, \\ \sum_{k=1}^{j} \binom{j-1}{k-1}(1-p_1)^k p_1^{j-k} P(Z=k), & j=1,\ldots,M-1, \\ 1 - \sum_{j=0}^{M-1} P(W_l = j), & j=M. \end{cases} \tag{11}$$

From this pmf we can easily compute $\overline{W}_l \equiv \overline{W}$ and $\sigma^2_{W_l} \equiv \sigma^2_W$.

To evaluate the number of days of work offered during a month having V work days, $Y \equiv \sum_{l=1}^{V} W_l$, we invoke the central limit theorem and approximate the cdf of Y as normal (Gaussian) with mean $V\overline{W}$ and variance $V\sigma^2_W$.

Indirect cost of temps is incurred when temps are paid for days not offered due to a shortfall in meeting the guarantee of G offered days per month, i.e., when $Y < MG$. The quantity $MG - Y$ is the number of days for which such 'lost time' pay is required. Therefore the expected number of days in a month that temps are paid due to not meeting the guarantee is

$$\overline{\mathrm{TI}} = E[\max(\{MG - Y\}, 0)]. \tag{12}$$

Invoking the central limit theorem, (12) can be closely approximated by

$$\overline{\mathrm{TI}} = \int_{-\infty}^{MG} (MG - y) f_Y(y)\ \mathrm{d}y \tag{13}$$

where $f_Y(y)$ is Gaussian with mean $V\overline{W}$ and variance $V\sigma^2_W$. Since the integral in (13) is a normal loss function, it is equal to

$$\overline{\mathrm{TI}} = L\left(\frac{V\overline{W} - MG}{\sqrt{V\sigma^2_W}}\right)\sqrt{V\sigma^2_W}, \tag{14}$$

where $L(z)$ is the standardized loss function (Tables of $L(z)$ are available in many books, e.g., [1]).

The expected monthly indirect cost of temps is

$$\overline{\mathrm{CT}}_{\mathrm{m}} = SC_{\mathrm{TE}}\overline{\mathrm{TI}} \tag{15}$$

The expected monthly total cost of the system is the sum of the expected monthly overtime cost, the expected monthly direct cost of temps, the expected monthly indirect cost of temps and the expected monthly cost of FTE's:

$$\bar{C}(M) \equiv S[VC_{\mathrm{OT}}\overline{\mathrm{OT}} + VC_{\mathrm{TE}}\overline{\mathrm{TE}} + C_{\mathrm{TE}}\overline{\mathrm{TI}} + NVC_{\mathrm{R}}]$$
$$= S[V\overline{\mathrm{OT}}(C_{\mathrm{OT}} - C_{\mathrm{TE}}) + VC_{\mathrm{TE}}\ E\ \max(Z, 0) + C_{\mathrm{TE}}\overline{\mathrm{TI}} + NVC_{\mathrm{R}}] \tag{16}$$

To find the optimal pool size we solve the problem

$$\min \quad \bar{\bar{C}}(M) \equiv (C_{\mathrm{OT}} - C_{\mathrm{TE}})V\overline{\mathrm{OT}} + C_{\mathrm{TE}}\overline{\mathrm{TI}}, \tag{17}$$
$$M = 0, 1, 2, \ldots.$$

To obtain the optimal pool size we can evaluate $\bar{\bar{C}}(M)$ for $M = 0, 1, \ldots$ and the optimal M is the one with smallest $\bar{\bar{C}}$. We conjecture that $\bar{\bar{C}}(M)$ is a unimodal function of M. This conjecture is based on our computational experience with many examples where always $\bar{\bar{C}}(M)$ was unimodal.

Finally, we compare our model to the extreme case where the system has a pool of size zero. In this case, following our discussion in Section 2.4,

$$\overline{\mathrm{OT}} = E\ \max(Z, 0) \tag{18}$$

and thus the expected overtime cost during the month is

$$\overline{\mathrm{OT}}_{\mathrm{m}} = (E\ \max(Z, 0)) \cdot S \cdot V \cdot C_{\mathrm{OT}}. \tag{19}$$

Since $M = 0$, $\overline{\mathrm{TE}} = \overline{\mathrm{TI}} = 0$ and therefore the expected monthly total cost of the system is just the sum of the expected monthly overtime cost and the expected monthly cost of FTE's:

$$\bar{C}(0) = S[VC_{\mathrm{OT}}E\ \max(Z, 0) + NVC_{\mathrm{R}}]. \tag{20}$$

Using expressions (16) and (20) and the first part of expression (9) $\forall M > 0$, $\bar{C}(0) - \bar{C}(M) > 0$, if

$$(C_{\mathrm{OT}} - C_{\mathrm{TE}})SV\,\overline{\mathrm{TE}} - SC_{\mathrm{TE}}\overline{\mathrm{TI}} > 0 \tag{21}$$

where $\overline{\mathrm{TE}}$ and $\overline{\mathrm{TI}}$ are calculated according to (9) and (14) (when $M > 0$). Expression (21) is quite intuitive. A pool of size $M > 0$ is better than a pool size of $M = 0$ if the saving in the direct cost of temps relating to the overtime cost is larger than the additional indirect cost of temps.

Example. Suppose that the number of FTE in our service facility is $N = 4$, the percentage of FTE who fail to show up to work is $p = 15\%$, the number of guaranteed days to temps, $G = 16$ and the number of work days in a month is $V = 26$. The cost of FTE's regular time is \$20 per hour, the cost of FTE's overtime is \$30 per hour and the cost of temps is \$16 per hour. It is also assumed that the pool size of temps is $M = 2$ and p_1 the percentage of temps who are not willing to show up to work on any given work day is also 15% and $S = 4$ hours.

Finally the PMF of D is:

d:	3	4	5	6
$P(D = d)$:	0.2	0.3	0.4	0.1

Since $L_1 = 3$ and $L_2 = 6$, Z can assume values between -1 and 6. From (4),
$P(\mathrm{OT} = 0 \mid Z = -1) = P(\mathrm{OT} = 0 \mid Z = 0) = 1$,
$P(\mathrm{OT} = j \mid Z = k) = 0$, $j = 0, 1, \ldots, k - 3$; $k > 2$ or when $j > k$; $k > 0$; or when $j > 0$; $k \le 0$,
$P(\mathrm{OT} = j \mid Z = k) = P(S_2 = 2) = (0.85)^2 = 0.7225$, $k = 3$, $j = 1$; $k = 4$, $j = 2$; $k = 5$; $j = 3$; $k = 6$, $j = 4$,
$P(\mathrm{OT} = k \mid Z = k) = 1 - P(S_1 \le M) = 0.0225$, $k = 1, 2, 3, 4, 5, 6$,
$P(\mathrm{OT} = 0 \mid Z = 1) = P(S_1 \le M) = 0.9775$,

$P(\mathrm{OT}=0 \mid Z=2)=P(S_2 \le M)=0.7225,$
$P(\mathrm{OT})=j \mid Z=k)=P(S_1 \le 2)-P(S_2 \le 2)=0.2550,\ j=1,\ k=2;\ j=2,\ k=3;\ j=3,\ k=4,\ j=4,\ k=5;\ j=5,\ k=6.$

From (3):

$P(Z=-1)=P(T=0)P(D=3)=0.1044,$
$P(Z=0)=P(T=0)P(D=4)+P(T=1)P(D=3)=0.2303,$
$P(Z=1)=P(T=0)P(D=5)+P(T=1)P(D=4)+P(T=2)P(D=3)=0.3389,$
$P(Z=2)=P(T=0)P(D=6)+P(T=1)P(D=5)+P(T=2)P(D=4)+P(T=3)P(D=3)=0.2311,$
$P(Z=3)=P(T=1)P(D=6)+P(T=2)P(D=5)+P(T=3)P(D=4)+P(T=4)P(D=3)=0.0793,$
$P(Z=4)=P(T=2)P(D=6)+P(T=3)P(D=5)+P(T=4)P(D=4)=0.0144,$
$P(Z=5)=P(T=3)P(D=6)+P(T=4)P(D=5)=0.0013,$
$P(Z=6)=P(T=4)P(D=6)=0.00005.$

Since $E(\max(Z,\ 0))=(0.3389)(1)+(0.2311)(2)+(0.0793)(3)+(0.0144)(4)+(0.0013)(5)+(0.00005)(6)=1.1036,$
$\overline{\mathrm{OT}}=1.1036-(2)(0.85)[0.2311+0.0793+0.0144+0.0013+0.00005]-(0.3389)(0.9775)=0.2172,$
$\overline{\mathrm{TE}}=1.1036-0.2172=0.8864.$

From (10):

$P(W_l=0)=P(Z \le 0)=0.1044+0.2303=0.3347,$
$P(W_l=1)=(1-(0.15))(0.3389)=(0.85)(0.3389)=0.288,$
$P(W_l=2)=1-0.3347-0.288=0.3773,$

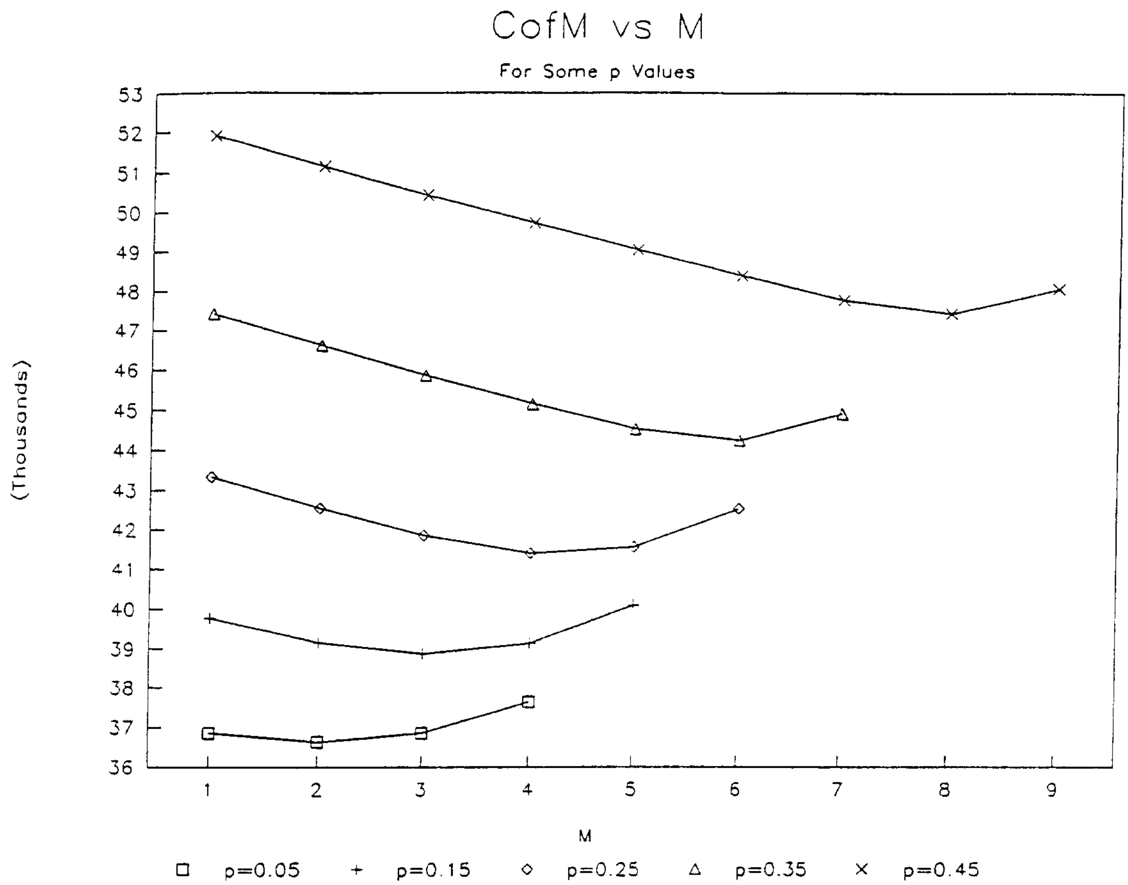

Figure 1: $\overline{C}(M)$ for p = .05, .15, .25, .35, .45

$\overline{W} = 0.288 + (2)(0.3773) = 1.0426$,
$\sigma_W^2 = 0.288 + (4)(0.3773) - (1.0426)^2 = 0.7102$.
Therefore

$$\overline{\mathrm{TI}} = L\left(\frac{(26)(1.0426) - (2)(16)}{\sqrt{(26)(0.7102)}}\right)\sqrt{(26)(0.7102)} = L(-1.1385)4.2971 = 5.1135,$$

and from (16),

$$\overline{C}(M) = (4)[(26)(30)(0.2172) + (26)(16)(0.8864) + (16)(5.1135) + (4)(26)(20)] = 10799.896.$$

Performing the same calculations for $M = 0, 1, 3, 4$ we obtain:
$\overline{C}(0) = 11763.23$, $\overline{C}(1) = 10968.42$, $\overline{C}(3) = 11040.22$ and $\overline{C}(4) = 12322$, and thus $M = 2$ is the optimal pool size for the example with optimal cost $\overline{C}(2) = 10799.896$.

For another example, suppose $N = 15$, $p = p_1 = 15\%$, $G = 16$ days, $C_R = \$20$, $C_{OT} = \$30$, $C_{TE} = \$16$, $S = 4$ hours, $V = 26$ days and

$$P(D = d) = \frac{e^{-15}15^d}{d!} \Bigg/ \sum_{d=0}^{25} \frac{e^{-15}15^d}{d!}, \quad d = 0, 1, \ldots, 25.$$

The total cost as a function of M for various values of p is shown in Figure 1.

As can be seen from Figure 1, as expected: (i) for fixed M increasing p (the fraction of FTE's that do not appear for work), result in increasing total cost of the system; (ii) when p is increasing the optimal pool size M is increasing as well; and (iii) for all cases in the figure $C(\overline{M})$ is unimodal.

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